Introduction to Dynamical Systems

Two-dimensional iterated maps (2): S/U manifolds.

October 1, 2015

Let the 2D map $f(x, y) = (x + y \mod 1, x + 2y \mod 1)$

Q1. Describe the unstable manifold of $(0, 0)$. Show that the stable and unstable manifolds meet at right angles.

Let the 3D map $f(x, y, z) = (x/2, y/2, 2z - x^2 - y^2)$. The origin is the only fixed point.

Q2. Show that the unstable manifold $U(0)$ is the z-axis.

Q3. Show that $S(0)$ is the paraboloid $\{(x, y, z) : z = \frac{4}{7}(x^2 + y^2)\}$.

Let the Henon map: $f(x, y) = (a - x^2 + by, x)$.

Q4. Compute f^{-1} (what is the condition on a and b, if any?)

Q5. Write a program that computes numerically the stable and unstable manifolds as follows:

- first, choose a saddle point p of the Henon map and compute its eigenvectors ; let V^u denote the vector tangent to the unstable manifold $U(p)$ associated with eigenvalue u ;
- then choose a point M on the line through V^u so that M and $N = f(M)$ are within 10^{-6} of p (if the eigenvalue $u < 0$, use f^2 instead of f, because then it is a flip-saddle) ;
- then apply f to the segment $J = [MN]$; this involves choosing a grid of points $M = M_0, M_1, M_2, \ldots, M_n = N$ along the segment J. Let $N_1 = f(M_1)$. The rule used here is that the distance $|N_1 - N|$ should be less than 10^{-3}. Otherwise move M_1 closer to M. Repeat this procedure when choosing each grid point (continue with N_2 so that $|N_1 - N_2| < 10^{-3}$, and so on).
- Using this method, calculate $f, f^2, \ldots, f^n$ of J (plot continuously as you progress).